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RAY CURVATURE AND REFRACTION OF WAVE PACKETS

by J. Ernest Breeding, Jr.

TECHNICAL REPORT NO. JEB-3

Department of Oceanography
Florida State University

September, 1978

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ABSTRACT

An expression for the ray curvature of a wave packet is derived which is suitable for use in wave prediction programs. The ray curvature of the wave packet is found to vanish if the packet direction becomes either perpendicular or parallel to the wave speed contours, assuming the wavelet direction is not parallel to the contours. For parallel water depth contours, this means that as a hydron moves into shoaling water refraction tends eventually to turn the hydron so that it is directed either perpendicular or parallel to the shoreline. The first case is similar to monochromatic waves. It is the result for hydrons which begin in deep water if the angle of incidence is between 0° and 74.8° with respect to the contour normal. However, for deep water angles of incidence equal to or greater than 74.8° the hydrons are turned and move parallel to shore in water of intermediate depth. The packet ray curvature approaches infinity as the wavelet direction, but not the hydron direction, becomes parallel to the wave speed contours. The result is total reflection of the waves. Total reflection occurs if a hydron is moving into deeper water and its initial direction exceeds a critical angle. At the reflection point the hydron direction becomes perpendicular to the water depth contours. Further, the hydron velocity goes to zero, which is consistent with a particle concept. As in quantum mechanics, the wave-particle duality is encountered.

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TABLE OF CONTENTS

	Page
Abstract	ii
Acknowledgments	iii
Table of Contents	iv
List of Figures	v
Introduction	1
Ray Curvature for Wave Packets	1
Packet Ray Curvature for Locally Parallel Wave Speed Contours	2
Properties of the Packet Ray Curvature and Snell's Law	3
Examples of Wave Packet Refraction-Hydrons	5
Waves from Deep Water	5
Reflection Points	7
Summary	10
References	13
Distribution List	14

LIST OF FIGURES

Page

Figure 1. Hydron trajectories for a 20 sec wave period for waves beginning in deep water 6

Figure 2. Monochromatic rays for comparison with the hydron trajectories in Figure 1.................6

Figure 3. The variation of the geometric group speed with kh for hydrons beginning in deep water 8

Figure 4. Hydron trajectories for a 20 sec wave period for waves which begin at an intermediate water depth 9

Figure 5. Variations of the wavelet direction γ, hydron direction θ, the speed $U = d\omega/dk$, and the geometric group speed $G = U \cos(\theta-\gamma)$ up to the reflection point for ray number 2 in Figure 4 11

Figure 6. The ratio of the hydron ray curvature to its absolute initial value as a function of the wavelet direction up to the reflection point for ray number 2 in Figure 4 12

Introduction

The conventional definition of group speed U has been defined as

$$U = d\omega/dk \tag{1}$$

where ω in the angular frequency and k is the wave number. This equation defines the speed of the group in the direction of the wavelet velocity. The geometric group speed G was defined by Breeding (1978) as

$$G = U \cos \phi \tag{2}$$

where

$$\phi = \theta - \gamma \tag{3}$$

The direction of movement of the wave packet is represented by θ, and the direction the wavelets move within the packet is specified by γ.

Breeding (1978) has shown that wave packets refract according to Snell's law with the geometric group velocity. This refraction law determines the wave path of constructive interference. At each point of the wave packet trajectory the wavelet direction is determined by Snell's law with phase velocity.

In this paper an expression is derived for the ray curvature of a wave packet and its properties are described. Examples of gravity water waves will be presented to show how G and U differ and to demonstrate the important features of wave packet refraction. In addition, wave packet and monochromatic trajectories will be compared for waves which begin in deep water.

Ray Curvature for Wave Packets

The ray curvature κ_v of a ray moving with phase speed v was derived by Munk and Arthur (1952) and Arthur, et al (1952) as

$$\kappa_v = \frac{d\gamma}{ds_v} = \frac{1}{v} \left(\sin \gamma \frac{\partial v}{\partial x} - \cos \gamma \frac{\partial v}{\partial y}\right) \tag{4}$$

where x,y are the Cartesian coordinates, γ is the direction of the ray with respect to the positive x-axis, and s_v is the arc length along the ray.

The ray curvature κ_G for the trajectory of a wave packet is given by

$$\kappa_G = \frac{d\theta}{ds_G} = \frac{1}{G}\left(\sin\theta \frac{\partial G}{\partial x} - \cos\theta \frac{\partial G}{\partial y}\right) \quad (5)$$

where θ is the direction of the packet ray with respect to the positive x-axis and s_G is the arc length along the ray.

Packet Ray Curvature for Locally Parallel Wave Speed Contours

In deriving an expression for the packet ray curvature the wave speed contours are assumed to be parallel. This assumption greatly simplifies the derivation. Further, it leads to an expression suitable for making calculations from which the important properties of the packet ray curvature can be determined. Accordingly, the xy-coordinate system is chosen such that the y-derivatives of v, U, and G are zero. The space derivative of G is then

$$\frac{dG}{dx} = \frac{dU}{dx}\cos\phi - U\sin\phi\left(\frac{d\theta}{dx} - \frac{d\gamma}{dx}\right) \quad (6)$$

Expressions for the space derivatives of θ and γ are obtained from (4) and (5) where

$$dx = ds_v \cos\gamma \quad (7)$$

$$dx = ds_G \cos\theta \quad (8)$$

Thus

$$d\gamma/dx = \left[(dv/dx)/v\right] \tan\gamma \quad (9)$$

$$d\theta/dx = \left[(dG/dx)/G\right] \tan\theta \quad (10)$$

After (9), (10), and (6) are substituted into (5) and the result is simplified, the packet ray curvature is found to be

$$\kappa_G = \frac{\left[(dU/dx)/U\right] + \left[(dv/dx)/v\right] \tan\phi \tan\gamma}{\csc\theta + \tan\phi \sec\theta} \quad (11)$$

This expression for the packet ray curvature can be used in wave prediction programs even if the wave speed contours are not everywhere parallel. From a practical viewpoint, it is only necessary that the wave speed contours be locally parallel in the vicinity of each ray point. Eq. (11) is used by Breeding et al 1978

in computing wave packet trajectories of gravity water waves. To use (11) the coordinate system is rotated at each ray point so that the y-axis is parallel with the water depth contours and the positive x-axis is directed toward deeper water.

Properties of the Packet Ray Curvature and Snell's Law

The ray curvature of a wave packet defined by (11) exhibits some very remarkable properties. It is assumed that v, U, and their derivatives are continuous and finite. However, under various conditions the trigonometric terms of the equation can become infinite or have indeterminate forms. The value of κ_G approaches zero as the wave packet direction θ becomes either parallel or perpendicular to the wave speed contours, provided the wavelet direction γ is not parallel to the contours. This means that given a sufficiently long path, refraction tends to turn the wave packet so that it is directed either parallel or perpendicular to the wave speed contours. If θ is neither parallel nor perpendicular to the wave speed contours, then κ_G approaches infinity as γ becomes parallel to the wave speed contours. In this case, due to the value of γ, the wave packet undergoes total reflection.

To determine the value of κ_G when there are indeterminate forms it is necessary to consider the variations of θ and γ as the indeterminate forms are approached. For example, (11) contains an indeterminate form when θ becomes perpendicular to the wave speed contours while γ becomes parallel to the contours. If γ approaches parallelism to the contours faster than θ approaches the perpendicular to the contours the value of κ_G becomes infinite.

The relationship between θ and γ due to refraction is clearly seen by integrating the ray curvature expressions (4) and (5). The y-derivatives being zero, integration of (5) leads to

$$(\sin\theta)/\left[U\cos(\theta-\gamma)\right] = C \qquad (12)$$

which is Snell's law for a wave packet where C is a constant. Snell's law with

phase velocity, which determines γ, is obtained by integrating (4). The cosine term in (12) can be replaced by the identity for the difference of two angles and the terms rearranged to yield

$$\tan \theta = (UC \cos \gamma)/(1 - UC \sin \gamma) \tag{13}$$

where the variation of θ appears only on the left side of the equation.

It is interesting to note that θ becomes zero if $\gamma = (2m + 1)(\pi/2)$ where m is an integer. Thus if the wavelet direction becomes parallel to the wave speed contours the wave packet direction becomes perpendicular to the contours. Further, note that $\theta = (2m + 1)(\pi/2)$ if $UC \sin \gamma = 1$. For this case the wave packet direction is parallel to the wave speed contours.

Snell's law can be used to derive an expression for $\cos(\theta-\gamma)$. Eq. (13) is substituted into the identity for $\tan(\theta-\gamma)$ and the result is simplified to obtain

$$\tan(\theta-\gamma) = (UC - \sin \gamma)/\cos \gamma \tag{14}$$

In terms of initial values, Snell's law with phase velocity can be written

$$\sin \gamma = v_r \sin \gamma_i \tag{15}$$

where $v_r = (v/v_i)$ and the subscript i denotes an initial value. Before refraction it is assumed that $\theta_i = \gamma_i$. Then

$$C = (\sin \gamma_i)/U_i \tag{16}$$

When (15) and (16) are substituted into (14), it is found that

$$\tan(\theta-\gamma) = \frac{(U_r - v_r) \sin \gamma_i}{(1 - v_r^2 \sin^2 \gamma_i)^{1/2}} \tag{17}$$

where $U_r = (U/U_i)$. This result can be transformed by the use of an identity to

$$\cos(\theta-\gamma) = \left[1 + \frac{(U_r - v_r)^2 \sin^2 \gamma_i}{1 - v_r^2 \sin^2 \gamma_i}\right]^{-1/2} \tag{18}$$

Eqs. (1), (2), and (18) provide a useful means of computing the geometric group speed.

Examples of Wave Packet Refraction - Hydrons

To demonstrate the properties of wave packet refraction, examples of gravity water waves will be considered. Gravity water waves are particularly suited as examples because of their highly dispersive nature. The term 'hydron,' suggested by Purser and Synge (1962) and Synge (1962), will be used to denote the wave packet of water waves.

Waves from Deep Water

In Figure 1 hydron trajectories are shown for waves beginning in deep water (water depth greater than one half the wavelength). The water depth contours are parallel. Initially $\theta_i = \gamma_i$ where each initial direction indicated on the figure is the angle between the hydron velocity vector and the normal to the depth contours. Regardless of the wave period, for deep water angles of incidence between 0^o and 74.8^o the hydrons follow paths such that the angles increase to the depth of the geometric group speed maximum (see Figure 3), then undergo a point of inflection, and then decrease shoreward. As a hydron approaches shore its direction becomes perpendicular to the wave speed contours and the packet ray curvature approaches zero. For deep water angles of incidence equal to or greater than 74.8^o the hydron trajectories turn and move parallel to shore in water of intermediate depth. As the hydron direction becomes parallel to shore the packet ray curvature tends to vanish; this is apparent in ray number 5.

For comparison, monochromatic rays are shown in Figure 2 for the same conditions considered in Figure 1. For large incident angles there is a striking difference between hydron and monochromatic trajectories. Whereas all the hydron rays do not reach shore all the monochromatic rays do. Note that the wavelet direction at each point along a hydron path in Figure 1 is the same as the direction of the corresponding monochromatic ray at the corresponding water depth.

It is interesting to compare the values of G and U when they differ due to

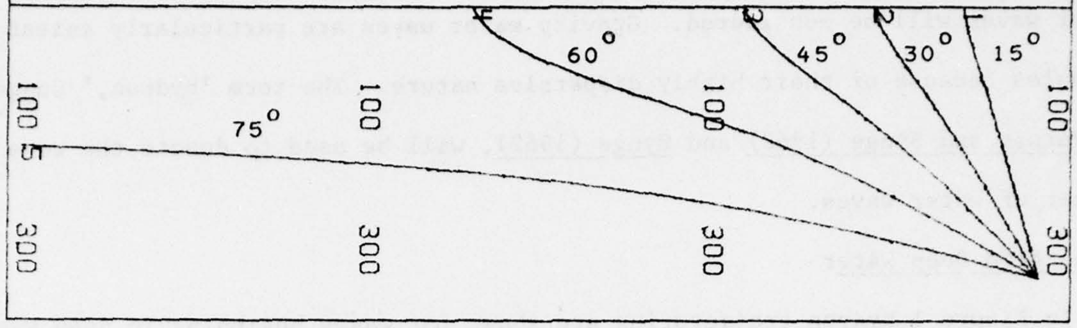

Figure 1. Hydron trajectories for a 20 sec wave period for waves beginning in deep water. The water depth contours are parallel, the scale of the plot is 1 cm = 4.87 km, and the sounding water depths are in meters. The initial hydron direction is shown for each ray and is the angle between the hydron velocity vector and the normal to the depth contours.

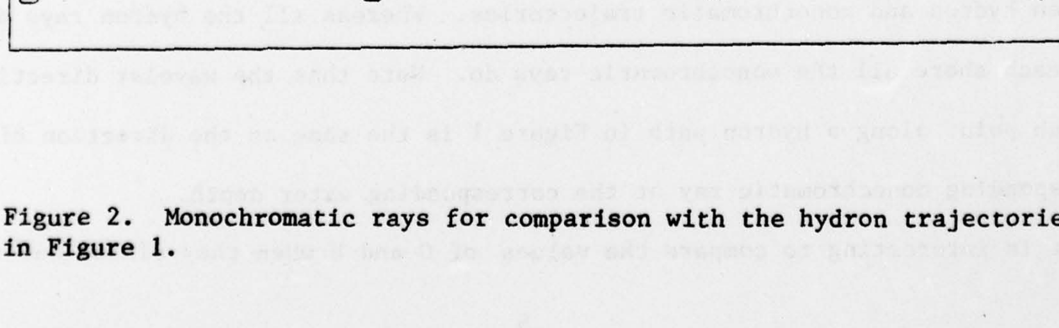

Figure 2. Monochromatic rays for comparison with the hydron trajectories in Figure 1.

refraction. As an example, for gravity water waves which begin in deep water it is found that

$$v_r = \tanh kh \tag{19}$$

$$U_r = \left(1 + \frac{2kh}{\sinh 2kh}\right) v_r \tag{20}$$

where h is the water depth. When (19) and (20) are substituted into (18) and the result is substituted into (2) it is found that

$$G = U\left[1 + \frac{(kh \sin \gamma_i \operatorname{sech}^2 kh)^2}{1 - (\sin \gamma_i \tanh kh)^2}\right]^{-\frac{1}{2}} \tag{21}$$

The ratio of the geometric group speed to its initial deep water value is presented for several incident angles in Figure 3. The initial hydron directions are defined as in Figure 1. The curve for $\gamma_i = 0^o$ is the same as obtained for the ratio of U to the value of U in deep water. The amount by which the other curves differ from it is a measure of the difference between G and U.

Inspection of Figure 3 shows for a given γ_i that the maximum of G/G_i occurs at a greater value of kh than does the minimum of G/U. An increase in γ_i causes a shift in both the minimum of G/U and the maximum of G/G_i to larger values of kh. Further, the maximum peak tends to get flattened out. The curve for $\gamma_i = 74.8^o$ is seen to stop abruptly at the maximum value of G/G_i.

When $\gamma_i = 30^o$ the maximum percentage difference of G from U is 2.70%. When $\gamma_i = 45^o$ the value is 5.91%, for $\gamma_i = 60^o$ it is 10.03%, and for $\gamma_i = 74.79^o$ the value is 14.27%.

Reflection Points

To obtain a reflection point it is necessary that the waves propagate into deeper water and that the initial direction of the hydron ($\theta_i = \gamma_i$) exceed a critical angle. The reflection point occurs at an intermediate water depth when, through refraction, the wavelets are turned parallel to the wave speed contours.

In Figure 4 two rays are shown in which the wave period is 20 sec and the

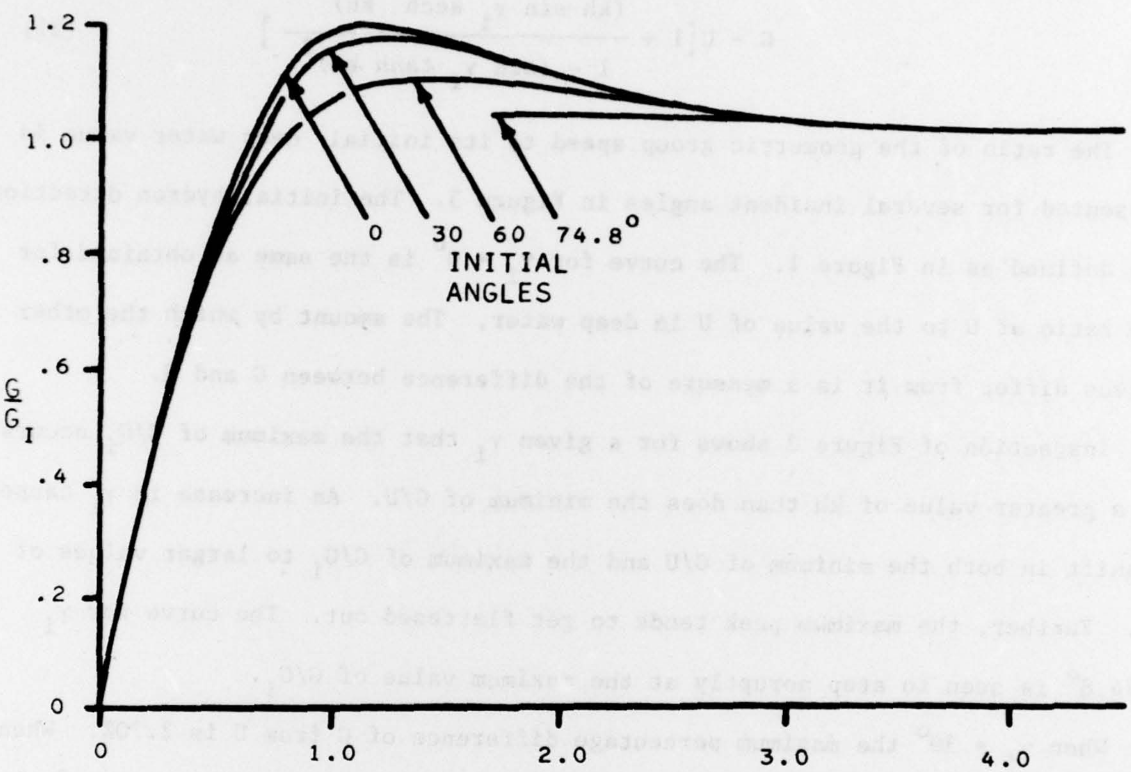

Figure 3. The variation of the geometric group speed with kh for hydrons beginning in deep water. The initial value of the geometric group speed is G_I. The hydron directions are defined as in Figure 1.

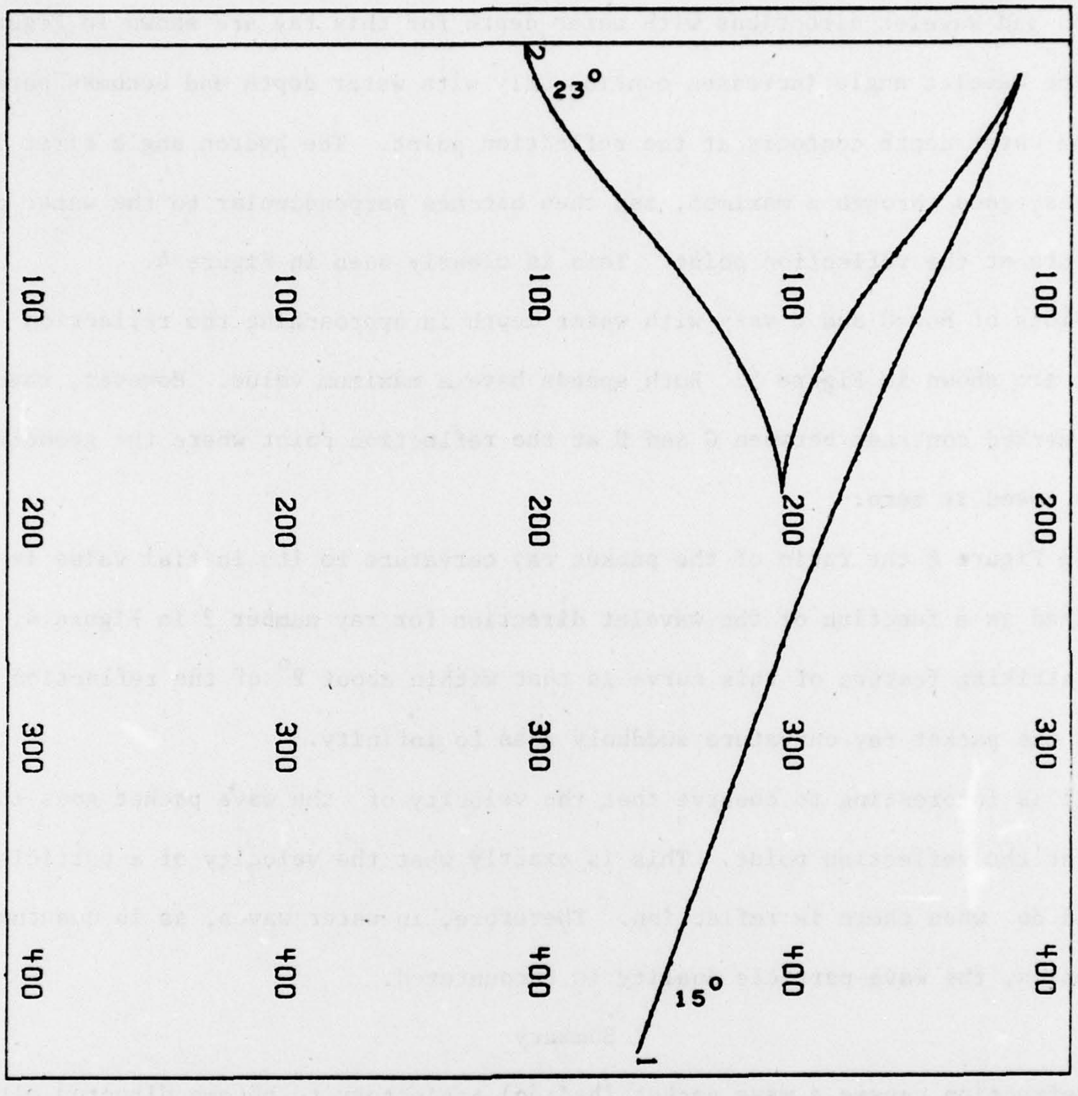

Figure 4. Hydron trajectories for a 20 sec wave period for waves which begin at an intermediate water depth. The scale of the plot is 1 cm = 3.10 km and the sounding water depths are in meters. The initial hydron direction is shown for each ray and is defined as in Figure 1.

initial water depth is 15 m. For this case a reflection point occurs if $\theta_i \geq 22.2^\circ$. Ray number 1 reaches deep water since $\theta_i = 15^\circ$. For ray number 2, $\theta_i = 23^\circ$, and a reflection point occurs at a water depth of 200 m. The variation of the hydron and wavelet directions with water depth for this ray are shown in Figure 5. The wavelet angle increases continuously with water depth and becomes parallel to the water depth contours at the reflection point. The hydron angle first increases, goes through a maximum, and then becomes perpendicular to the water depth contours at the reflection point. This is clearly seen in Figure 4.

Plots of how G and U vary with water depth in approaching the reflection point are shown in Figure 5. Both speeds have a maximum value. However, there is a marked contrast between G and U at the reflection point where the geometric group speed is zero.

In Figure 6 the ratio of the packet ray curvature to its initial value is sketched as a function of the wavelet direction for ray number 2 in Figure 4. The most striking feature of this curve is that within about 2° of the reflection point the packet ray curvature suddenly goes to infinity.

It is interesting to observe that the velocity of the wave packet goes to zero at the reflection point. This is exactly what the velocity of a particle should do when there is reflection. Therefore, in water waves, as in quantum mechanics, the wave-particle duality is encountered.

Summary

Refraction causes a wave packet (hydron) trajectory to become directed either parallel or perpendicular to the water depth contours. In either case the packet ray curvature will vanish. For hydrons propagating toward deep water, if the initial direction exceeds a critical angle total reflection occurs. At the reflection point the wavelet direction becomes parallel to the wave speed contours, the hydron direction becomes perpendicular to the contours, the geometric group velocity goes to zero, and the packet ray curvature becomes infinite.

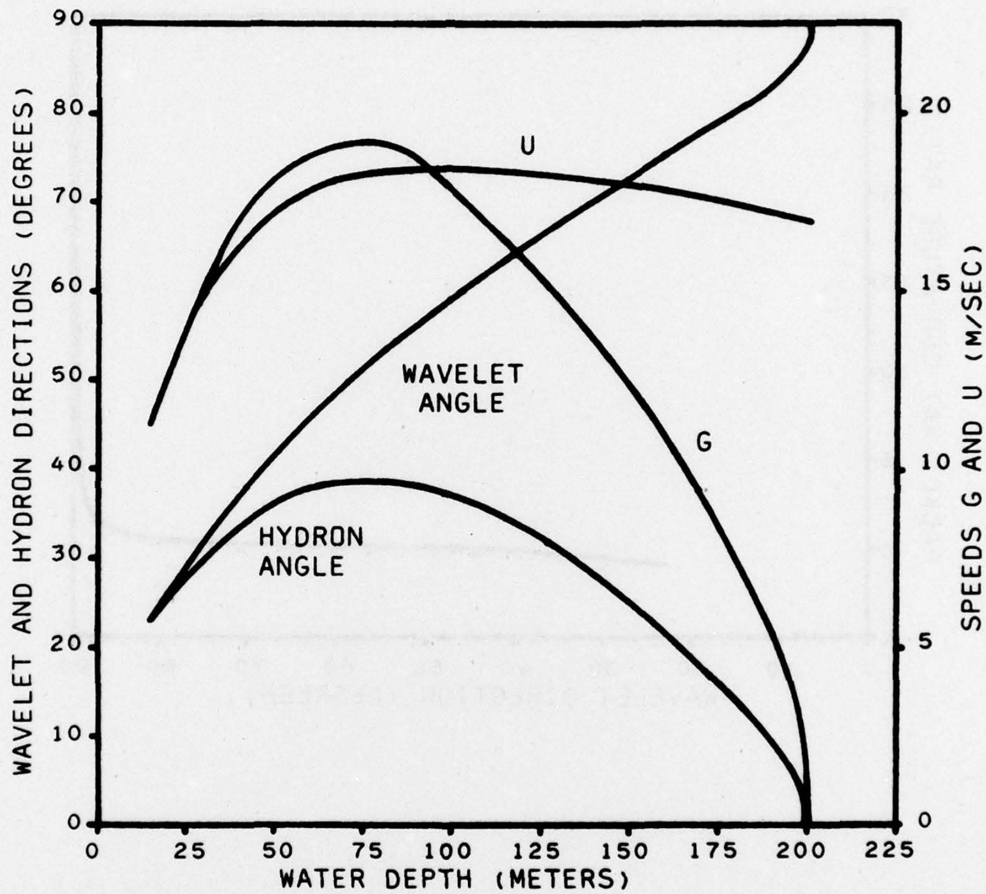

Figure 5. Variations of the wavelet direction γ, hydron direction θ, the speed $U = d\omega/dk$, and the geometric group speed $G = U \cos(\theta-\gamma)$ up to the reflection point for ray number 2 in Figure 4.

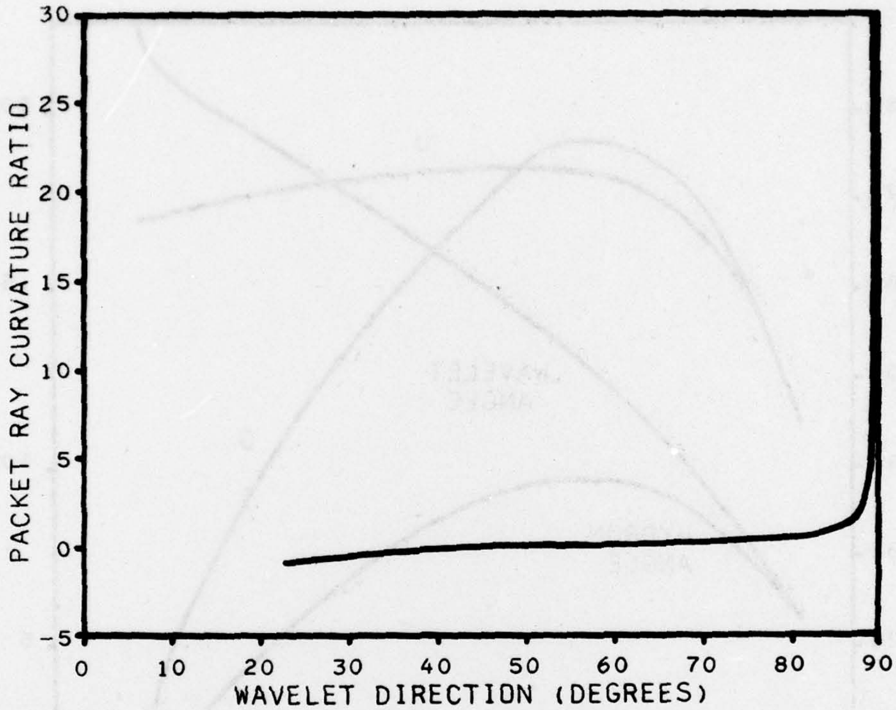

Figure 6. The ratio of the hydron ray curvature to its absolute initial value as a function of the wavelet direction up to the reflection point for ray number 2 in Figure 4.

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An expression for the ray curvature of a wave packet is derived which is suitable for use in wave prediction programs. The ray curvature of the wave packet is found to vanish if the packet direction becomes either perpendicular or parallel to the wave speed contours, assuming the wavelet direction is not parallel to the contours. For parallel water depth contours, this means that as a hydron moves into shoaling water refraction tends eventually → next pg.

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to turn the hydron so that it is directed either perpendicular or parallel to the shoreline. The first case is similar to monochromatic waves. It is the result for hydrons which begin in deep water if the angle of incidence is between 0 deg and 74.8 deg with respect to the contour normal. However, for deep water angles of incidence equal to or greater than 74.8 deg, the hydrons are turned and move parallel to shore in water of intermediate depth. The packet ray curvature approaches infinity as the wavelet direction, but not the hydron direction, becomes parallel to the wave speed contours. The result is total reflection of the waves. Total reflection occurs if a hydron is moving into deeper water and its initial direction exceeds a critical angle. At the reflection point the hydron direction becomes perpendicular to the water depth contours. Further, the hydron velocity goes to zero, which is consistent with a particle concept. As in quantum mechanics, the wave-particle duality is encountered.